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CONCEPTUALIZING TEACHERS' UNDERSTANDING OF STUDENTS' MATHEMATICAL LEARNING BY USING ASSESSMENT TASKS*

ABSTRACT. The study was designed to support teachers on conceptualizing their understanding of students' learning by the use of assessment tasks. A school-based assessment team consisting of the researcher and four third-grade teachers teaching in the same school was set up as a learning context of supporting teachers in developing assessment tasks integral to instruction. The assessment tasks along with students' responses to the task, classroom observations, interviews, routine weekly meetings, teachers' weekly reflective journals, and students' responses to the assessment tasks were the data collected in the study. The teachers' views of using assessment tasks and the generation of assessment tasks were developed in the course of the study. In the process of generating assessment tasks, teachers improved their awareness of students' various solutions and learning difficulties to a specific problem, their awareness of the importance of developing students' critical thinking, and their awareness of where students need to make a remedial instruction.

KEY WORDS: assessment tasks, professional development, students' responses

INTRODUCTION

The reform efforts of many countries call for knowledge of students' common conceptions and misconceptions about the subject matter which is essential for teaching (Australian Education Council, 1991; MET, 1993; NCTM, 2000). The current reforms recommend that effective teaching refers to knowing to ask critical questions to help students reflect on their own thinking, create mathematics tasks and analyze students' responses in order to make ongoing instructional decisions, and stimulate classroom discourse so that students are clear about what is being learned. These skills reveal the importance of knowing students' learning in playing the role of effective mathematical teaching (MET, 1993).

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There are increasing studies in teacher education focusing on enhancing knowledge of students' learning about the mathematics topics teachers will teach (Dawson, 1999; Fennema, Carpenter, Franke, Levi, Jacobs & Empson, 1996; Simon, 1995). These studies suggest that one major way to improve mathematics instruction is to help teachers understand the mathematical thought processes of their students. Thus, the principles of designing professional development programs paid more attention to the importance of knowledge of students' learning (Loucks-Horsley, Hewson, Love & Stiles, 1998). The studies on knowledge of students' learning began either by offering a research-based model of a children's mathematical thinking or by participating in workshops that guided teachers to reflect on students' thinking to help teachers learn to understand students' thinking. The Cognitively Guided Instruction project directly enables teachers to understand their students' thinking by providing research-based models of children's mathematical thinking in well-defined content domains (Fennema et al., 1996). Some researchers (e.g., Cobb, Wood, Yackel & McNeal, 1993) did not directly work with children's thinking; instead, they encouraged teachers to develop constructivist pedagogy by having them participate in workshops that guided their reflection about students' thinking. Cobb and his associates claim that a teacher has to interpret classroom events and that the teacher is a learner concerning children's understanding. They found that teachers constructed knowledge about learning in general but made only tentative comments about individuals, treating these as working hypotheses rather than as assessments that could contribute to decision making of instruction. However, little research on assessment in mathematics education has focused on the teacher-assessor as an active constructor of knowledge about students' mathematical knowledge.

The use of assessment tasks is another possible way to create the opportunity for conceptualizing teachers' knowledge of students' learning, since classroom assessment allows teachers to gain insight into students' conceptual knowledge and reasoning while working on the tasks. This conception of assessment portrayed in the Principles and Standards for School Mathematics treats every instructional activity as an assessment opportunity for teachers and a learning opportunity for students (NCTM, 2000). The document advocates the significance of assessment as integral to instruction in the classroom as well as assessing students' learning to inform teachers as they make moment-by-moment instructional decisions about students' work in the classroom. This indicates that teachers are likely to learn about students' learning from students' work performing on the assessment tasks used in daily lessons. Thus, the use of assessment integral to instruction becomes the focus of the study for increasing teachers' knowledge of students' mathematical learning.

The Use of Assessment Tasks for Conceptualizing Teachers' Understanding Students' Learning

Webb (2004) makes a distinction between internal and external assessment in that internal assessment involves teachers. The significance of the role teachers playing in classroom assessment is aligned with the assessment triangle referred by Pellegrino, Chudowsky & Glaser (2001). The triangle includes three interrelated elements: cognition, observation, and interpretation, as the key elements when designing and implementing classroom assessment, placing teachers at the core of classroom assessment. Cognition refers to teachers' conceptions of how students represent knowledge and develop competence in mathematics. Observation refers to the opportunities teachers create to elicit students' responses. Interpretation is the method teachers use to make sense of students' work and performance. In this triangle, observation is related to the tasks teachers select or generate, while interpretation is related to teachers' interpretation and feedback of students' work. According to the triangle model, generating assessment tasks and analyzing students' responses are two vital elements of teachers designing and implementing classroom assessment.

Many researchers suggest that teachers' conceptions of mathematics, pedagogical content knowledge, and conceptions of how students learn mathematics all have a significant influence on teachers' classroom practices (Franke, Carpenter, Levi & Fennema, 2001). With respect to assessment, teachers' conception of how students learn mathematics influences not only how they interpret student work but also the content and nature of feedback they provide (Morgan & Watson, 2002). However, Webb worked with K–12 teachers to support change in their classroom assessment practices as they implemented a reformed curriculum. Webb found that teachers' conception of student learning did not translate into changes in the selection and scoring of assessment tasks in ways that would support the assessment of student leaning. Teachers often preferred using sets of skill-oriented tasks that matched classroom assignments and a percent-based scoring system rather than generating assessments for promote reasoning, communication, non-routine problem solving, and connection. Teachers who continued to use conventional assessment practices while implementing reformed curriculum were challenged by the discrepancy between their personal assessment of what students knew form classroom interactions and the results they obtained from their more conventional routines for assessing, scoring, and grading student work (Romberg, 2004). Webb's studies suggest that teachers' conceptions of student learning influences their classroom practices, but do not warrant changing their classroom assessment.

One undoubted reason is that teacher choice of skill-oriented tasks may simply be seen as deference to a more practical approach. The other reason is that the design of open-ended assessment tasks and analyzing students' responses are complex and challenging work for the teachers who are used to the traditional test. The complicated work engaging in generating an open-ended task includes formulating better reflection questions, marking student responses with comments instead of scores, and selecting tasks to make students thinking more visible.

Helping teachers toward a view of assessment integral to instruction seems to require new experience in classroom assessment that allows students to demonstrate what they are able to do and requires support from collaborative communities of assessment practice. One of the ways to acquire such learning experience is by providing teachers with an arena for learning about assessment. Frequently, teachers have never experienced such learning environments in their academic learning. Therefore, teachers need such experience in their professional development program. Training teachers in the design and use of assessment tasks has also been proposed as a means of improving the quality of assessment tasks (Senk, Beckmann & Thompson, 1997). Studies of teachers working together over time have suggested that teacher professional dialogue is the key to ensuring equality in teachers' assessments (Clarke, 1996).

These two challenges can only be achieved by establishing an assessment team who are mutually supportive by providing the opportunities for dialogue on critical assessment issues related to instruction. Thus, the paper reports more on how a teacher's professional development program provides teachers support for generating open-ended tasks and analyzing students' responses to the tasks.

The use of assessment does not warrant the improvement of teachers' understanding about students' learning; it depends on the presentation of assessment tasks and the way in which students are to respond (Heuvel-Panhuizen & Gravemeijer, 1993). Conventional skill-oriented tasks reveal only the results and tell nothing about the students' strategies and too little information is obtained about the progress of instruction. Such tasks make it difficult to diagnose students' mathematical difficulties. As an alternative to eliminate the failure of skilled-oriented tasks, open-ended tasks are often considered to be a valid way to allow students to show what and how they are able to accomplish something. According to De Lange (1995), a task that is open for students' processes and solutions is a way of stimulating students' high quality thinking. Moreover, it is evidenced by Amit & Hillman's (1999) study that teachers improve their ability to follow students' thought processes through using open-ended tasks. These studies imply that students' thinking processes are more likely to be performed on the open-ended tasks. However, the tasks used in the studies are conduced by teacher educators or researchers instead of classroom teachers (Amit & Hillman, 1999; Heuvel-Panhuizen, 1996). There is limited attention paid to the assessment tasks for inquiring about student's understanding conducted by teachers themselves based on their ongoing instructional activities. The open-ended tasks referred to in this study were not only to help teachers gain a better understanding of what students are able to do but also to create the opportunity for students learning high quality thinking, such as communicating, problem posing, problem solving, and critical thinking. Due to the limitation of the space, the paper merely describes the support teachers gained from a professional development program and their understanding about students' learning teachers developed which then informed their teaching. The improvement of students' high quality learning is not the focus of the paper.

Two Frameworks of Teacher Professional Development

Current reform asks teachers to make complex changes (MET, 1993). To achieve this vision requires both changes in school culture and teachers as learners (Franke et al., 2001). Educational reform efforts are doomed to fail if emphasis is on developing specific teaching skills, unless teachers' cognitions are taken into account (Cooney, 2001). Moreover, teachers' cognition is largely practical and situated (Jaworski, 1998; Krainer, 2001). Thus, teachers' learning is grounded in their interactions with students. Similar ideas are developed in a framework for designing professional development by Rhoton and Bowers (2001) and an interconnected model in a document entitled Teachers Professional Growth by the Teacher Professional Growth Consortium (TPGC) (1994, The University of Melbourne, unpublished). The interconnected model developed by the TPGC suggests that change occurs through the mediating processes of reflection and enactment in four distinct domains encompassing the teacher's world: personal, practice, consequence, and external domains. Personal domain refers to teacher's knowledge, beliefs, and attitudes. Practice domain refers to professional experiment, such as the use of collaborative group work, of extended mathematical investigations prompted by a task linked to a real world context, and the use of student self-assessment. Consequence domain referring to salient outcome includes teacher control, student motivation, and student development of new mathematical ideas. External domain includes external sources of information or stimuli such as professional publications and conversations with colleagues. This model identifies the mediating processes of reflection and enactment as the mechanisms by which change in one domain leads to change in another. The mediating processes of reflection and enactment are linking the four domains. The four domains and two mechanisms of the interconnected model are considered to serve as the guideline of enhancing teachers' knowledge growth.

Rhoton & Bowers (2001) suggest that a four-step cycle including: set goals, plan, do, and reflect needs to be considered to guide the work of professional developers. There are several areas in which there is substantial information that can influence choices that designers make as they develop a plan: learners and how they learn, teachers and how they teach, the nature of mathematics as a discipline, and knowledge grounded in research as well as in the experienced professional developers. Their framework, which includes four inputs – context, knowledge and beliefs, critical issues, and strategies for professional learning – is critical to consider when designing professional development. They suggest that the factors which constitute the context could include: the needs and nature of the students; the backgrounds and needs of the teachers; the resources available and the degree of community support; the organization and current demands of the schools. They claim that neglecting critical issues may block the success of the professional development. The issues are building a professional culture, developing leadership, and supporting standards. There are numerous strategies that can be combined in different ways at different times to enhance teacher learning. Selecting among strategies and knowing how to combine them at a different point throughout a program depends on the knowledge of best practice, the particular content, and the analysis of critical issues described earlier.

These two frameworks were embedded into the teacher professional development program reported in the paper. Enhancing teachers' knowledge is the goal of the program. To this end, assessment integral into instruction serves as a learning strategy for teachers. Learning contexts for teachers such as external professional support cannot be neglected. Reflection is one of the mechanisms to enhance teachers' knowledge growth.

The Study

Theoretical Framework of the Teacher Development Program

In order to implement the current curriculum into practices for success, teachers need to be more versatile in using instructional strategies for facilitating students' competencies. Such changes require that teachers have long-term support and adequate resources. Therefore, practical and effective professional development programs are needed in order to move the curriculum reforms ahead. A number of strategies have been explored in the program reported in this study to improve teachers' knowledge of students' thinking. The learning strategies included student-focused classroom observation, case construction, design of assessment tasks, analysis of students' responses, and students-focused curriculum investigation. The researcher has examined the effect of each strategy on knowledge of students' learning (Lin, 2002, 2005). This paper is not intended to report the effect of each learning strategy; instead, it is to describe in detail the effect of one of the strategies – generating assessment tasks and analyzing students' responses to the tasks – on knowledge of students' thinking.

The theoretical framework of the teacher professional development program consists of learning goals, learning strategies, and learning contexts. The interventions of the program are given in Figure 1.

The goals of the program include: (1) enhancing the rethinking of mathematics teaching in classrooms; (2) fostering teachers' awareness of children's learning; (3) supporting teachers as they begin to incorporate their new vision of a learner-centered approach to teaching; and (4) improving teachers' reflective ability. As depicted in Figure 1, the main learning goal increased teachers' knowledge of students' learning in mathematics. Critical reflection was also considered as a goal to be learned in the program. Figure 1 reveals that the area of each knowledge domain was unequal at each grade level. Teachers teaching at the primary grade level paid significant attention to students' learning, while teachers teaching at the upper grade level drew more concern on mathematics content. The complexity of mathematics increased by grade level made it more difficult for teachers. Knowledge of learners' cognition referred to knowing students' learning difficulties, error patterns, and prior knowledge, and how students think and learn in specific mathematics contents. Pedagogical content knowledge refers to knowing to create the mathematics classroom as a place for thinking and learning, to design worthwhile mathematical tasks fitting into curriculum, and to analyze mathematical goals.

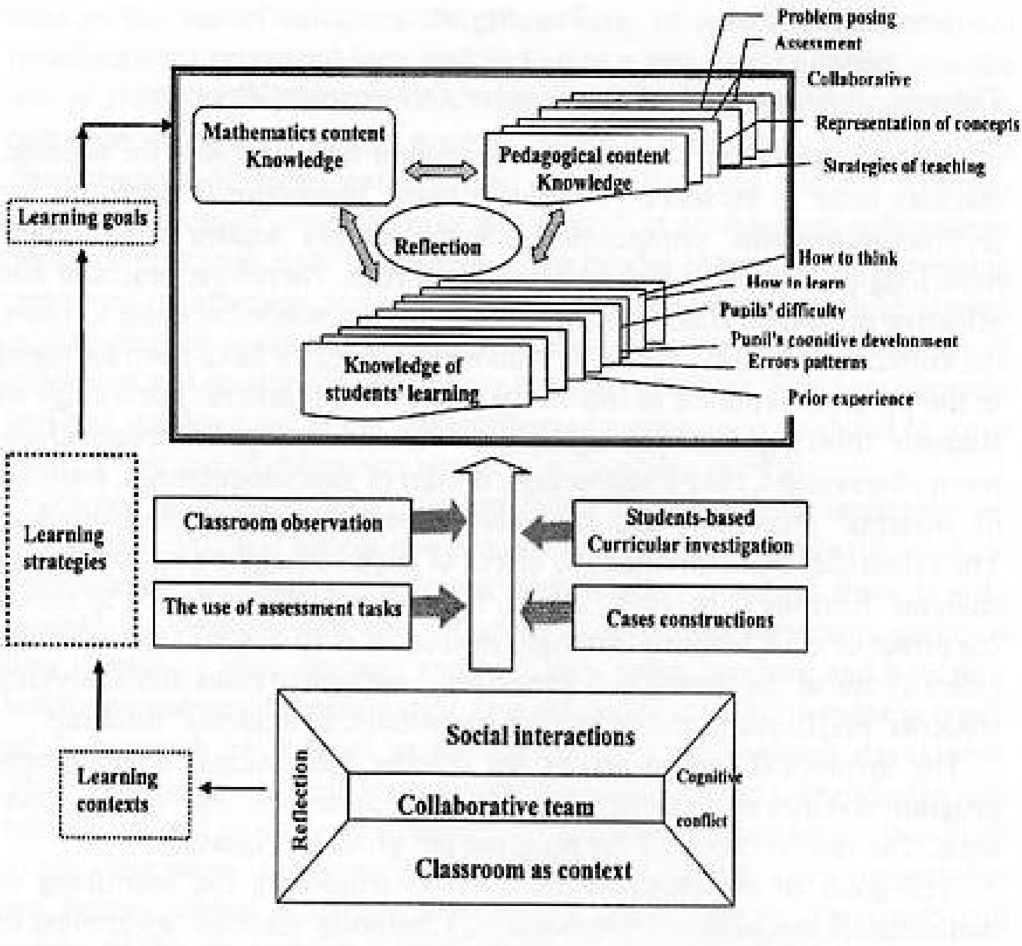


Figure 1. Theoretical framework of the teacher professional program.

The rationale of the framework has been influenced to some extent by Vygotsky's (1978) social interaction. Social interaction constitutes a crucial source of constructing mathematical knowledge. As such, mathematics learning is an interactive and constructive activity. Thus, within the program, a collaborative school-based professional team consisting of the researcher and the teachers from the same school was set up for providing professional dialogues stemmed from classroom practices.

In addition to social interaction, reflection and cognitive conflict were considered as the other two mechanisms of enhancing teachers' knowledge growth. Reflection is viewed as the cornerstone of the program as it is prerequisite to recognize and change routine behavior. Teacher becoming a reflective practitioner has been given considerable emphasis in the literature on professional development programs (Krainer, 2001; Lerman, 2001; Lin, 2002; Wood, Scott-Nelson & Warfield, 2001). Cognitive conflict produces a refining of one's cognitive structures. Cognitive conflict causes an imbalance proving the internal motivation for an accommodation. With this in mind, routine weekly meetings and classroom observations were considered to create the opportunity for the teachers to become reflective practitioners.

Mathematics classrooms and weekly routine meetings for the school-based professional team are two social contexts of teachers mutually supporting the teacher education program. Classroom observations created opportunities for communication to their colleagues what teachers learned from classrooms. Routine weekly meetings created the opportunities for discussing the situations that occurred in one teacher's classroom by comparing them to others.

The Assessment Practices in Mathematics Classroom (APMC) Project

The APMC project as part of the teacher professional development program was designed to help teachers implementing assessment integral to instruction into classroom practice. To achieve this aim, teachers were encouraged to use students' responses to tasks as a way of gathering the information about students' thinking processes, strategies, and their mathematical understanding. Open-ended assessment tasks served as the writing prompts for assessing students' thinking since (1) we wanted to establish a better means of communication among students, parents, and teachers about mathematics learning taking place in classrooms, and (2) we were looking for a better way to assess each student's entire learning process by writing about mathematics. The open-ended tasks were considered an informal way of assessing what and how individual students learned from daily lessons. The use of the assessment tasks referred to in the study was encouraged to be a way of formative evaluation of students' mathematics learning. The percentage of the use of the assessment integral to instruction as part of students' summative assessment depended on how teachers valued the use of assessment tasks.

The generation of mathematical tasks as the core of the APMC project considered several concerns which include: (1) supporting a method of assessment that allows students to demonstrate their strengths; (2) stimulating students to make connections for mathematical ideas; (3) generating good tasks that do not separate mathematical processes from mathematical concepts; and (4) generating the assessment tasks for inspecting what and how students learned from daily lesson. Designing the assessment tasks based on daily classroom lessons and analyzing students' responses to the tasks, as an aspect of assessment integral to instruction, was defined as the use of assessment tasks reported in the paper. The following questions focused the investigation.

1. What were the teachers' conceptions of using assessment tasks under the support of a teacher professional development program?
2. What were the effects of using assessment tasks on conceptualizing teachers' understanding of students learning?

METHOD

Participants

A school-based "assessment team" consisting of the researcher and four third-grade teachers was set up to discuss the assessment issues occurring in the classroom. A particular school was selected because the teachers were willing to learn, the administers provided support and it had ever been previously involved in the program. The four female teachers, given pseudonyms for confidentiality, are referred to as Yo-Yo, Mei-Mei, Jen-Jen, and Ying-Ying. Years of teaching experience in primary schools range from 1 to 16 years. Ying-Ying has only one year teaching in elementary school and four years teaching in secondary school. Mei-Mei has only one year as a qualified teacher and two years as a substitute teacher. Yo-Yo has 15 years of classroom teaching and one year participating in the researcher's previous study, and Jen-Jen has 13 years of teaching experience.

The researcher played different roles in facilitating, probing, and giving feedback to teachers. The role of the researcher was not to provide ready-made tasks for teachers to use, but rather to create the opportunity for teachers to sit together to design creative assessment tasks. This was done because the researcher believed that teachers' knowledge of classroom assessment is constructed during the process of designing tasks via their social interactions. The researcher was expected to be the carrier of theoretical perspective, while the four teachers were expected to be the practitioners of implementing assessment integrated with instruction into the classroom.

The Contexts of the Study

Routine Weekly Meetings. School-based assessment team routine weekly meetings centered on generating written exploratory tasks and students' responses to the tasks were teachers' regular participation. The weekly meetings were to create the learning community by generally providing support as the teachers learned to design assessment tasks and to analyze students' responses to the assessment tasks. The teachers were required to bring their students' responses to the weekly meetings to share with others.

To achieve the purpose of the use of tasks, the following questions were supplied to nudge the teachers to rethink: What did you expect to learn about your students from this task? Were you satisfied with your students' performance on the task? Did you really gain what you want to gather? After reviewing the students' work, each teacher was required to report in the meeting what they learned from the students' responses.

Third-Grade Classrooms. Third-grade classrooms were the other primary context for teachers' learning to design written exploratory assessment tasks and analyze students' responses. There were two reasons for selecting teachers from the same grade to participate in the study. First, the teachers teaching the same mathematics topics were confronted with similar pedagogical problems. Similar mathematical content lent itself readily as a focal point when the teachers met together after observing each other's lessons. Secondly, similar pedagogical issues drew attention and concern from each teacher, leading to in-depth discussions.

To provide ongoing support to the teachers, build relationships and collect data, we observed a classroom together as a means of learning more about student thinking and teachers' assessment practices. To develop the instructor's knowledge about generating assessment tasks, the instructor was encouraged to design at least one possible assessment task to examine if her students understood the activities engaged in the instruction. The lessons of the teachers were scheduled to be observed in turn. The teachers observed a classroom lesson once a month on a Monday morning and had a meeting lasting for three hours immediately after the observation. Although lessons were not observed every week, meetings were held weekly allowing the teachers to learn from one another's assessment practices. To understand teachers' practices of using assessment tasks, I encouraged them to write their reflective journal after each weekly meeting.

Support for Teachers on Generating Written Exploratory Assessment Tasks

To achieve the goal of assessment integral to instruction, the assessment tasks of the study were not conducted prior to instruction but generated from the activities students engage in during a daily lesson. This is a new challenge for the teachers.

To support the teachers in designing good tasks (meaning that the tasks capture students' curiosity and invite them to speculate and to pursue their hunches), I encouraged them to follow the following four principles. The first is that written exploratory tasks from the daily lessons given to students each time were less than two. The second is that, due to labor-loaded work, each teacher generated a task once a week and then shared it with her same grade level colleagues. I encouraged them to be mutually supportive in generating good novel tasks. The third is that each teacher was required to analyze students' work prior to bringing students' responses to the weekly meeting, and the fourth is that each teacher was required to report in public what she learned from the tasks and from students' responses. The assessment tasks were generated and revised from the teachers' professional dialogues in weekly meetings. Thus, the teachers were frequently involved in observing teaching together, dialoguing as a group, and reflecting on the quality of tasks.

In addition, the interviews were conducted by the researcher twice during the year to collect teachers' responses to the practice of generating assessment tasks. The first interview was in the middle of the study and the other was conducted at the end of the study. Each teacher was interviewed individually for an hour in a quiet room of the primary school with the following two questions: (1) How would you value the use of assessment tasks for your teaching and students' learning? (2) How did the use of assessment tasks help you to understand your students' learning and inform your mathematics teaching?

Data Collection and Analysis

The data for this study were gathered from several sources: 20 videotaped classroom observations, 8 interviews, 30 audio-recorded weekly meetings, teachers' reflective journals, and 75, 30, 51, and 41 copies of written exploratory assessment tasks from Yo-Yo, Mei-Mei, Jen-Jen, and Ying-Ying, respectively, and their students' responses to the tasks. The assessment tasks the teachers generated were analyzed by categorizing for understanding of the teachers' learning about assessment.

On one hand, we carefully examined each assessment task in weekly meetings throughout the school year as to whether the context of problems or the students' inquiries covered in the task occurred in real teaching, since assessment integral to instruction is the essential principle of the study. To validate the use of each assessment task the teachers conducted, each assessment task was encouraged to be applied to other classrooms. Eleven rubrics of assessing the written tasks the teachers conducted were developed by the researcher and the four teachers, as shown in Table I. Seven of them had to do with the nature of the tasks and four of them were concerned with the purpose of the tasks. Each assessment written task was assigned by the teachers working in a group into the categories according to the purpose of the task and also according to the degree of usefullness for each category of the nature of the task (Table I).

On the other hand, while sitting together the teachers were asked to identify each of the written assessment tasks by its purpose in the last two weekly meetings of the study. The analysis of tasks referred to in the study only focused on the purpose of the tasks. They identified the tasks as four purposes: problem-posing, communicating mathematics, problem solving, and critical thinking.

To enhance teachers' knowledge of students' learning, the teachers brought their students' work to share with the group. The group analysis of students' responses to the tasks frequently took the major part of each weekly meeting. The analysis of students' response to the tasks requiring them to create a word problem was based on six rubrics as seen in Table II.

The analyses of students' responses to the written tasks requiring students to communicate mathematics, problem solving, and make a critical analysis of others' solutions were based on the variety of student strategies. The teachers analyzed the whole classroom performance item

TABLE I

The rubric of assessing the written tasks the teachers generated

	Features of the tasks teachers conducted	not good	medium	good
Nature of the tasks	1. The task is concise and precise.			
	2. The difficulty of the task is medium.			
	3. The task is relevant to students' daily life.			
	4. The context of the task is interesting.			
	5. The task is inquiry-oriented.			
	6. The task is creative.			
	7. The task focuses on the procedure rather than the result.			
Purpose of the tasks	8. The task is for facilitating students' problem posing			
	9. The task contributes to mathematic understanding.			
	10. The task contributes to mathematical communication.			
	11. The task contributes to critical thinking.			

TABLE II

The rubric of assessing students' responses to the written tasks requiring students to create a word problem

Features of the problem students created
1. The problem is ill-structured.
2. The problem is irrelevant to mathematics.
3. The problem is not reasonable.
4. The information given in the problem is insufficient.
5. The problem is well-structured.
6. The problem provides extra information.

by item according to students' various solutions. Students' responses to the tasks to be used for illustration as examples were transcribed as closely as possible to the students' exact words.

Triangulation of the data used multiple sources, multiple methods, and multiple analysts. The data collected from weekly meetings throughout the entire year, individual interviews at two different times, and individual teacher's weekly reflective journals were for validating the data of teachers' value of using assessment tasks. To validate the rubric of each task assigned, each teacher first categorized individually each item and then made a consensus through the group discussion.

The data was analyzed using a ground-theory approach, as described by Strauss & Corbin (1994). In this approach, the researcher is the primary instrument of data collection and analysis, applying inductive methods and striving to derive meaning from the data. In keeping with this approach, there were no predetermined criteria or coding system in the analysis. To document teachers' learning from the use of assessment tasks, transcripts of interviews and weekly meetings, observations, and teacher's reflective journals, a procedure was used in which all documents were reviewed and comments were made in the margins. All comments included in the margin of the observation sheet, reflective journals, or transcription sheets were a part of ongoing analysis.

When patterns were detected and analyzed, two themes emerged relevant to teachers' conception of using assessment tasks. These were the value teachers gave to the use of tasks and teachers' preference for use of assessment tasks. Three themes emerged showing the effect of using assessment tasks on conceptualizing teachers' understanding of students' mathematical learning which included: improving teachers' awareness of students' various solutions and learning difficulties to a specific problem, improving teachers' awareness of the importance of developing students' critical thinking, and improving teachers' awareness of where students need to make a remedial instruction.

RESULTS

Teachers' Conception of Using Assessment Tasks

Teachers' preference for the use of assessment tasks and their value in the use of assessment tasks generated from daily ongoing teaching are described as in this section.

Teachers' Preference for the Written Assessment Tasks. The numbers of tasks the teachers generated indicated their preference for the written tasks in the study. There were 197 assessment tasks with written prompts compiled from the four teachers throughout the school year. The number of assessment tasks in each category that each teacher generated is summarized in Table III. The results show that the number of each category of assessment tasks each teacher generated varied from their teaching experience and involvement in the researcher's previous studies. The results indicate that Yo-Yo, with more years of teaching and research experience, conducted more tasks than the novice teacher Ying-Ying did. Moreover, the tasks Yo-Yo generated were interesting, creative, inquiry-oriented, and were described concisely and precisely.

The number of the tasks requiring students to create a word problem, requiring students to communicate mathematics, requiring students to make sense of their understanding, and requiring students to justify others' solutions were 45, 20, 85, and 47, respectively. The results indicate that the teachers generated assessment tasks to clarify what students learned in the classroom with the highest frequency, while they generated tasks for assessing students' communication of mathematics with the lowest frequencies. In addtion, while looking at the tasks shown in Table IV in accordance with the purpose of the tasks, we found that the use of assessment tasks in the first half of the school year was less frequent than in the second half of the school year.

The problem-posing tasks requiring students to formulate story problems were categorized into four subcategories. The tasks were given numbered sentences, pictorial representations, mathematical languages, and students' various solutions to ask students to create a word problem.

TABLE III

Frequencies of the rubric of assessing the written tasks the teachers generated

Features of the tasks teachers conducted		Not good				Medium				Good			
		Yo	Mei	Jen	Ying	Yo	Mei	Jen	Ying	Yo	Mei	Jen	Ying
Nature of the task	1. The task is concise and precise.	0	0	2	10	9	8	14	4	66	22	35	27
	2. The difficulty of the task is medium.	0	0	0	0	2	2	31	21	73	28	30	10
	3. The task is relevant to students' daily life.	0	0	0	15	11	11	3	12	64	45	40	14
	4. The context of the task is interesting.	0	0	0	18	21	13	26	13	54	17	25	10
	5. The task is inquiry-oriented.	2	4	4	26	3	28	25	9	48	13	23	12
	6. The disk is creative.	0	0	5	19	23	22	23	10	52	8	23	12
	7. The tasks focused on assessing the procedure rather than the result.	0	0	0	20	0	18	19	8	75	12	32	13
Purpose of the task	8. The task is for improving students' problem posing.	0	0	0	0	0	1	3	2	17	8	7	8
	9. The task contributes to critical thinking.	0	0	2	0	8	5	7	6	22	10	13	12
	10. The task contributes to mathematical communication.	0	0	0	3	2	1	1	5	7	3	4	15
	11. The task contributes to critical thinking.	0	0	0	3	3	1	2	6	16	6	10	9

TABLE IV

The number of assessment tasks each teacher generated

Participants	First semester					Second semester					
	PP	CA	PS	CT	Total	PP	CA	PS	CT	Total	Total
Yo-Yo (15, 1)	5	3	12	6	26	12	6	18	13	49	75
Mei-Mei (1, 0)	1	1	5	1	8	3	3	10	6	22	30
Jen-Jen (13, 0)	1	1	9	2	13	11	4	13	10	38	51
Ying-Ying (1, 0)	4	0	6	0	10	8	2	12	9	31	41
Total	11	5	32	9	57	34	15	53	38	140	197

Yo-Yo (15, 1): Yo-Yo has 15 years of teaching experiences and 1 year involved in the previous study the researcher investigated.
PP: problem-posing tasks.
CA: communication tasks.
PS: problem-solving tasks.
CT: critical-thinking tasks.

The tasks for assessing students' communication were categorized into five subcategories. The tasks were given the expression of one's own mathematical thinking by using various representations, understanding others' mathematical thinking, understanding others' questioning to a specific solution, compensating for others' thought when it is incorrect, and finding fault in unreasonable thought. The problem-solving tasks ask students to use representations to present the mathematics concept. The prompts of asking students to write a summary of what they learned in a classroom lesson belong to the problem-solving tasks. Critical thinking tasks were sorted in accordance with four elements: making-up the incomplete problems students generated, repairing the incorrect problems teachers provided, completing students' incomplete solutions, judging one's logical thinking process. The examples of each category of assessment tasks are given in Figure 2.

The progress the teachers made in using assessment tasks resulted from their better understanding about the value of the written assessment tasks and their gradually improving skill in generating a task integral to instruction. An excerpt from Mei's reflective journal is evidenced as follows.

At the beginning, I asked my students to write the key ideas either by drawings or written words that we have been taught today. However, they did not know what should be written and their parents could not give them help due to without their

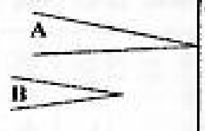
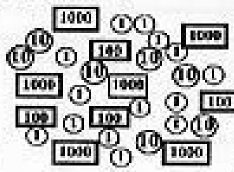
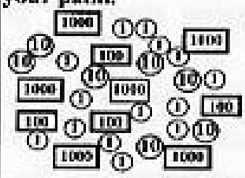
Problem-posing Task	Communication Task	Problem-solving Task	Critical-thinking Task
Chinese version: 老師上課時出了一題數學，你能從下面兩個同學的作法猜猜看老師出的題目是什麼？ 廷豪：30+40=70 5+9=14 70+14=84 晟晏：35+40=75 75+5=80 80+4=84 老師的題目是：	**Chinese version:** 波波用三角板畫了兩個角，但是他無法判斷那一個大？請你告訴他，那一個角大？並說明原因。 	**Chinese version:** 太平洋SOGO百貨公司開幕大放送，將於頂樓往下灑錢，只要你手掌可以覆蓋到一半以上的錢都是你的喔，試試看你可得多少？  你得了___張千元，___張百元，___個十元，___個一元，共得___元。	**Chinese version:** 夢蓉想知道丁班上的公佈欄和黑板哪一個大？他用資料夾排，結果需要70個才能將布告欄覆蓋，用報紙排，結果需要20張才能將黑板覆蓋。 夢蓉說：資料夾用的比報紙多，所以布告欄比較大。 請問您同意夢蓉的說法嗎？為什麼？
English version: A problem was given by the teacher. Can you guess what the problem was in accordance with the following two students' solutions? Ting-Ho's : 30+40=70 5+9=14 70+14=84 Cheng-Yung's : 35+40=75 75+5=80 80+4=84 The problem the teacher gave was :	**English version:** Po-Po drew two angles by using a triangle ruler. He is unable to compare which of the angles is bigger? Would you please to tell him which one is bigger? Give him a reason why it is. 	**English version:** Money spread from the sky to the floor as the celebration of the SOGO opening ceremony. The money belongs to you once the half size of each money at least was covered by your palm.  You gained:___pieces of thousands, _____pieces of hundreds, ______pieces of tens,________pieces of ones. In total:	**English version:** Mon-Jong wants to compare the size of bulletin board and blackboard in a classroom. He used 70 profiles to cover bulletin board. Alternatively, he used 20 pieces of newspaper to cover the blackboard. Mon-Jong concluded that the blackboard is bigger than the bulletin board, since the number of profiles is more than the number of newspaper. Do you agree Mon-Jong's reason? Why?

Figure 2. Four categories of assessment tasks generated from daily lessons.

participation in classroom teaching. The limited words students wrote were merely relevant to their interesting or impressive events, such as a funny action happening in the classroom, but did not involve mathematics learning. Accordingly, I frequently received parents' phone calls at night to ask me about the activities we engaged in today's teaching.

Mei-Mei became used to applying the assessment tasks as part of her instruction in the second half of the school year. Her mastering skill in generating assessment tasks was expressed in the interview data conducted at the end of the school year as follows.

R: How often did you use the written assessment tasks in your classroom?

Mei: It is very frequent to be conducted while the objective of a lesson is on developing students' concept rather than on practicing routine problems. It is not necessary to use written prompts as tasks while the lesson goes through on practicing routine problems, since it can be achieved by students workbooks accompanied by textbooks students used.

R: Did you feel comfortable with using the written tasks for students?

Mei: It does not bother me very much. I was not comfortable with creating assessment tasks associated with instruction until Yo shared her writing task initiated from classroom teaching to clarify if her students made sense of the number sequence from 1,000 to 2,000. I am competent with figuring out one or two written prompts immediately after the classroom teaching. I am not anxious with the use of assessment tasks, because they are tied to classroom teaching altogether.

Teachers' Significance Value of Using Written Assessment Tasks. It is noted that the teachers valued the use of written assessment tasks for changing their teaching approach toward the mathematics community, becoming more sensitive to students' learning, as the medium to communicate students' learning with parents, and as a powerful cognitive tool to collect students' various solutions.

The Use of Assessment Tasks for Making the Change of Instruction. It is found that the use of assessment tasks moved their classrooms more towards mathematics communities and away from a collection of individuals, and shifted their teaching toward emphasis on process of thinking and away from mechanistic answer-finding. For instance, Yo-Yo shifted her classroom towards a mathematic community by giving students daily activities supporting inquiry and by motivating student participation. She stated in a weekly meeting:

Due to the need of offering students successful and grateful experience of writing assessment tasks, it pushed me to figure out designing attractive and creative teaching for my students. Otherwise, I seldom played games with my students in my teaching as usual.

Jen-Jen expressed in her journal her commitment to the value of analyzing students' responses to the assessment tasks by making her instruction change:

After reviewing students' responses to the assessment tasks we generated, I found that assessment is tied highly to instruction. The assessment tasks generated by those who are newcomers such as both Ying-Ying and Mei-Mei taking an instructor-centered teaching approach, did not stimulate students' multiple and in-depth thoughts; rather, a body of meaningless algorithms. As we observed in the classroom, they neglected the emphasis on the process of students' thinking and did not invite students' various solutions. As a consequence, their students did not and unable to answer with multiple methods to the assessment tasks. In a nutshell, students' in-depth thinking could not be developed well, even though we implemented the assessment tasks into the classroom if we do not change our instruction toward students-centered approach.

After reviewing the assessment task Ying-Ying produced from a multiplication lesson, Yo-Yo described her positive feedback to the use of assessment tasks in her journal as follows:

Professor Lin shared with us Ying-Ying's instruction toward focusing on the learner's thinking in today's meeting. Ying-Ying started to change her teaching based on the assessment task we discussed today. As we have seen her tasks conducted in the past month, Ying-Ying always gave her students close-up assessment tasks that reflected her traditionally instructional approach toward mathematics as a set of algorithms. I am happy with the use of assessment tasks, since it leads us to have the insight of the significance of developing students' learning with understanding in our teaching.

The Use of Assessment Tasks to Help Teachers Becoming Sensitive to Students Learning. The teachers agreed that they became more sensitive to students' learning through analyzing students' responses to the assessment tasks. For instance, the task "*Drawing pictures to represent the meaning of* $4 \times 5 \times 3$" which Jen-Jen generated to examine if students make sense of the meaning of multiplication. Three teachers, including Jen-Jen, sorted students' written work into four different categories as shown in Figure 3, while Yo-Yo sorted them into three categories. During a meeting Yo-Yo explained that:

Solution 1 is identical to solution 3. The students using solution 1 represented $5 \times 4 \times 3$ as 5 strips of 4 and then 3 groups of 5 trips in total; similarly, the students who used solution 3 represented $5 \times 4 \times 3$ as 5 columns of 4 in each pile and then three piles in all. As I

Task: ***Drawing pictures to represent the meaning of 4x5x3.***

Students' responses:

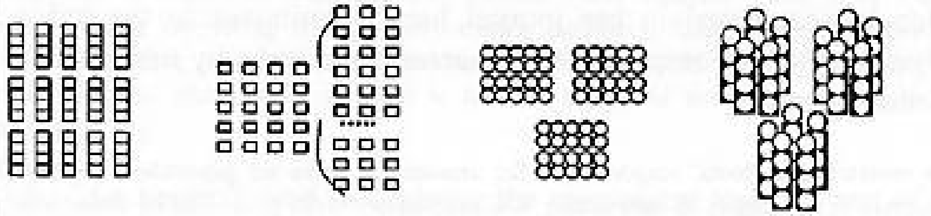


Figure 3. Students' various responses to the task.

observed the classroom, Shing-Ming used solution 2 to solve the problem. The first step he made was to represent 5x4 as shown in the left part of solution 2 in Figure 3. Then he rearranged the representation into a column of 20. What he showed is three columns of 20 altogether.

The researcher mentioned that there were no students using a three-dimensional representation like a rectangle to represent $5\times4\times3$. If students used the three-dimensional representation, then teaching multiplication was ready to move forward to the associative property of multiplication. At the end of the weekly meeting, Jen-Jen reflected the use of the assessment task. She replied that:

I have never known that the assessment task is a powerful cognitive tool to collect students' various solutions. The analysis of students' various responses through the group work is very helpful for me to understand what students learned and how students understood. It therefore expanded to me an insightful perspective of students' thinking. Before participating in the assessment project, I only focused on how many students answered correctly when I graded students' work, even less I did not give students such kind of open-ended task before. The discussion on students' responses to the assessment task helped me to inform further instruction in the property of associative under multiplication.

Assessment Tasks as a Medium for Communicating with Parents. The teachers valued students' responses to the assessment tasks as a medium for communicating students' learning with parents and to inform parents to know the new trend of currently innovative mathematics teaching. The following statement is an excerpt from Yo-Yo's reflective journal.

The assessment tasks informed my students' parents to know what students learned in my classes, to know the help they need to offer to their kids, and to know the importance of daily activities children engage in every day. The assessment tasks I generated were connected closely with students' daily activities. Parental interruptions to my teaching are gradually decreasing through the use of assessment tasks, since they perceived the diverse thinking of students as the focus of my teaching. In the beginning of the school year, the parents blamed me on spending too much time on inquiring students' various thoughts instead of teaching algorithms and mastering them. Two months later, my students' parents perceived that their kids' achievements and attitudes toward mathematics learning have been improved very much while they reviewed their children's profiles collected from students' performance of the assessment tasks. The parents are satisfied with their children on the improvement of mathematical achievement, such as, mathematical thinking becoming flexible, imagination becoming rich, oral and writing communication skills becoming better, and their positive attitudes of learning mathematics. Close to the end of the year, most of the parents appraise me and have the confidence with my teaching. Moreover, they expect me to participate continually in Professor Lin's research in the following year.

Effects of Using Assessment Tasks on Conceptualizing Teachers' Understanding of Students' Mathematical Learning

Improving Teachers' Awareness of Students' Various Solutions and Learning Difficulties. The following task was administered to the students who had learned multiplication with one-digit numbers for multiplicand and multiplier. As we observed, Yo-Yo paid much attention to make the distinction between $6 \times 5 = ( )$ representing 5 groups of 6 and $5 \times 6 = ( )$ representing 6 groups of 5,[1] and between $0 \times 8 = ( )$ and $8 \times 0 = ( )$ in her lesson. However, she was not sure if individual students understood the meaning of multiplicand 0 and 1. Right after the lesson, she generated the task by asking students to formulate word problems represented as $1 \times 5 = ( )$ and $0 \times 7 = ( )$.

Yo-Yo brought students' written responses from the task to the weekly meeting for the teachers to analyze. Table V shows the analysis of 35 students performing this task. It shows that 33% of the students did not give a clear statement of the multiplicand 1.

Four students' responses to the task demonstrated in Figures 4–7 reveal their diverse cognitive development in multiplication learning. From the unclear description of the problem it is seen that Wu had trouble with the meaning of multiplicand 1, as shown in Figure 4. The number "1" of $1 \times 5 = ( )$ represents "clocks in each class" instead of "a clock in each class." Taking advantage of the improper word problem, the researcher suggested that Yo-Yo should bring the problem to the next day's lesson to ask students to repair it.

The next day, Yo-Yo acted as though she needed help by asking students, "*Is it [There are 5 third-grade classes in Din-Pu school. There*

TABLE V

Analysis of 35 students performing written tasks requiring them to create a word problem

Features of the problem students created	Percentage of students	
	$1 \times 5 = ( )$	$0 \times 7 = ( )$
1. The problem is ill-structured.	3%	0
2. The problem is irrelevant to mathematics.	0	0
3. The problem is not reasonable.	6%	25%
4. The information given in the problem is insufficient.	33%	0
5. The problem is well-structured.	58%	75%
6. The problem provides extra information.	0	0

are clocks in each class. How many are clocks there altogether?] right? Could you help me to repair it so that it can be solved?" As observed, students were readily devoted to repairing the improper problem.

Students' responses to the task also showed that some of the third-graders had difficulty understanding the multiplicand 1 by using pictorial representation. Although Sue better understood the meaning of multiplication by giving a word problem, she had difficulty with using pictorial representations such as □, □, □, □, □ to represent "*5 cows each producing a bucket milk*" as shown in Figure 5. Likewise, Wu had difficulty modeling the problem by using ○×○○○○○=○○○○○ standing for 1 × 5 = 5, as shown in Figure 4.

Learning from students' responses to the assessment task, Yo-Yo perceived that Tsai's cognition of multiplication still stayed at "repeated addition" as shown in Figure 7, while Liu had better understanding of

Task: *If you were a teacher, how would you give your students a word problem represented by the number sentence (1) 1x 5= ( ) (2) 0x 7= ( ). Write it in words and represent it by drawings.*

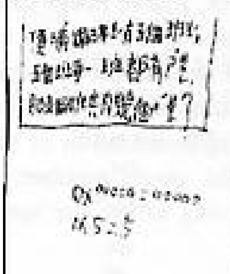
Chinese version: 	**Chinese version:** 	**Chinese version:** 	**Chinese version:** 
English version: There are 5 third-grade classes in Din-Pu school. There are clocks in each class. How many are clocks there altogether?	**English version:** A cow produces a bucket of milk. How many buckets of milk do 5 cows produce totally?	**English version:** A supermarket sells eggs. There are no eggs in each carton. How many eggs did I have to pay for 7 cartons of eggs?	**English version:** I cannot get allowance. My elder brother cannot get allowance. My elder sister cannot get allowance. My younger brother cannot get allowance. My younger sister cannot get allowance. Dad cannot get allowance. Mom cannot get allowance. Because we are naughty. There is no money we get altogether.

Figure 4. Wu's writing. *Figure 5.* Sue's writing. *Figure 6.* Liu's writing. *Figure 7.* Tsai's writing.

multiplication. Nevertheless, Mei-Mei criticized the problem Liu created as shown in Figure 6 as not being reasonable in a real situation, because selling the cartons with no eggs in a market was not reasonable in daily life. Mei-Mei commented that the problems students created were more interesting to them than those she had ever posed in the classroom. For instance, the problem a student created was that "*A policeman has a gun with no bullets. He fired 7 times aiming at a robber. How many (times) was the robber hit?*" became a good source for her further instruction.

As usual, the researcher required the instructor Yo-Yo to reflect on the assessment task at the end of the weekly meeting by asking "*what did you learn from the tasks and students' responses to the task?*" She answered that:

I learned that the well-designed task affords me the opportunities to learn about my students' understanding. The problem-posing task requiring students to create word problems is a powerful task to assess students' understanding. Moreover, through reviewing my students' responses to the task with your analysis, I know more about individual student's learning as well as more generally about students' difficulties.

In addition, Jen-Jen with 13 years of teaching experience reflected on the assessment task in her weekly reflective journal as follows:

Although I taught multiplication several times in my teaching carrier, I have never used the problem-posing tasks to assess whether students understand the concept of multiplication. I did not focus my multiplication teaching on the distinction between the multiplicand 1 and 0 as well as multiplier 1 and 0 and general multiplication, until we reviewed Yo-Yo students' responses today. Instead, I focus on the memorization of the rule of multiplication as: 1 multiplied by a given number is the given number; 0 multiplied by a given number always is 0, and vice versa. I am going to use the same assessment task teaching tomorrow.

To understand the practice of using assessment tasks, I asked each teacher to review the profile of students' responses to the tasks at the end of the study for evaluating students' learning throughout the entire school year. In a weekly meeting, Yo-Yo stated the effect of using assessment tasks on recognizing students' diverse learning styles:

...I also learned my students' various learning styles from their responses to assessment tasks. For instance, Hsio-Wei is a low achiever in mathematical learning. He still needed an assistance of hands-on material to count one by one with the number more than 10 when he entered the third grade. Number is an abstract symbol to him. I realized that he is a learner of typically physical operation based on his responses to the tasks I generated. He always needed the help by drawing pictures to make sense of number(s). He made considerable progress after I reviewed his profile of assessment tasks.

Facilitating Teachers' Awareness of the Importance of Developing Students' Critical Thinking. Jen-Jen has shifted her classroom toward a mathematical community and away from a set of individuals by the use of assessment tasks. She created an opportunity for students to learn the communication of mathematics and thinking mathematics critically. This is evidenced by the assessment task she generated and her lessons we observed. The task as seen in Figure 8 generated from a lesson was administered to examine whether each student perceived others' solutions discussed in Jen-Jen's classroom. As observed, the emphasis Jen-Jen placed on how a problem was solved is important to its answer. Students' various solutions were open to questioning and elaboration from others in her teaching. The questions she asked commonly in her classroom included *"Do you think it is true?," "Why do you think so?," "How could you prove that?,"* etc. Jen-Jen observed that the climate that students supported one another's ideas became a feature of her classroom. The following task displaying three students' methods was generated from her lesson (as shown in Figure 9).

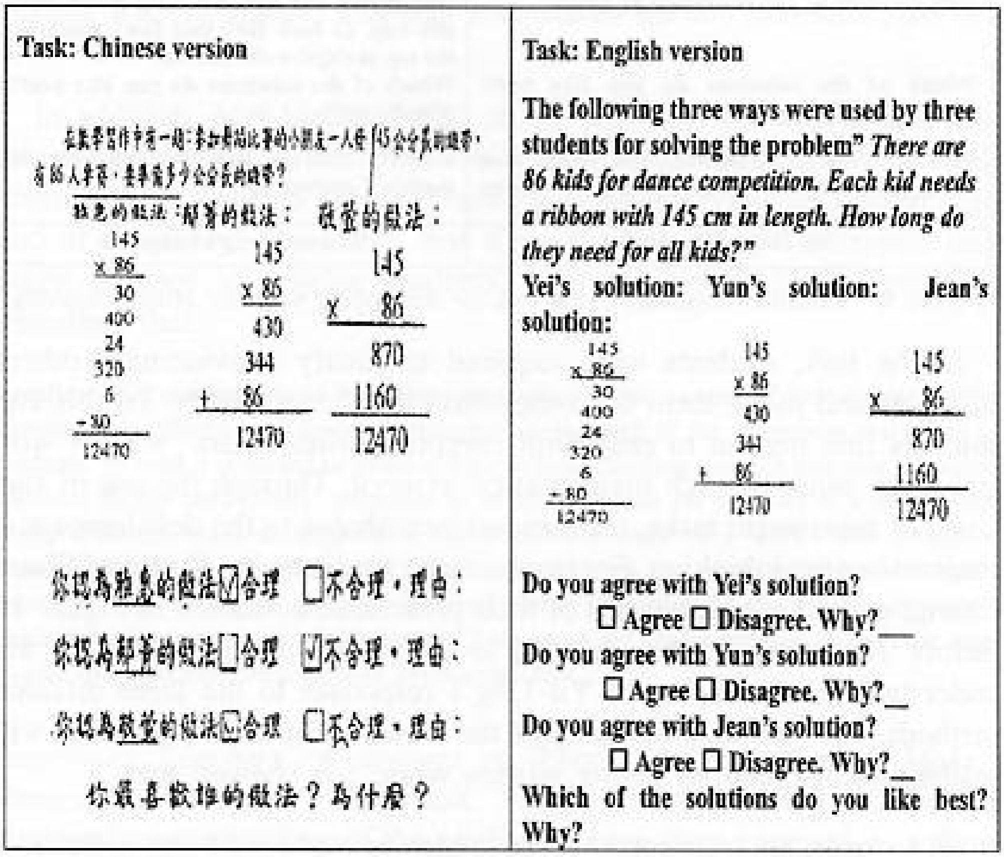


Figure 8. Task of developing students' critical thinking.

Task: The following three ways were used by three students for solving the problem" ***There are 86 kids for dance competition. Each kid needs a ribbon with 145 cm in length. How long do they need for all kids?"***

Yei's solution:	**Yun's solution:**	**Jean's solution:**
145 × 86 30 400 24 320 6 + 80 12470	145 × 86 430 344 + 86 12470	145 × 86 870 1160 12470

Do you agree with Yei's solution? **☑ Agree ☐ Disagree. Why?___**	**Do you agree with Yei's solution?** **☑ Agree ☐ Disagree. Why?___**
Because Yei used 5 times 6, 5 times 80, 4 tens times 6, 4 tens times 80, and one hundred times 6, and one hundred times 80. Then, she added them up altogether.	***Because I can understand it easily.***
Do you agree with Yun's solution? **☑Agree ☐ Disagree. Why?__**	**Do you agree with Yun's solution?** **☑ Agree ☐ Disagree. Why?__**
Because Yun used 86 multiplied by 5 first and then followed by 86 multiplied by 4 tens and 86 multiplied 1 hundred. Afterwards, she added them up together.	***The method needs to use mental calculation as well. It is a little for me.***
Do you agree with Jean's solution? **☑ Agree ☐ Disagree. Why?__**	**Do you agree with Jean's solution?** **☐ Agree ☑ Disagree. Why?__**
Because Jean used 145 multiplied by 6 and then followed by 145 multiplied by 8 tens.	***Because Jean's method used the bottom multiplying the top. We have never used this way. In both Yei's and Yun's methods, the top multiplies the bottom.***
Which of the solutions do you like best? Why? *Jean's*.	**Which of the solutions do you like best? Why? Yei's**
Because Jean's is shorter and understand easily. It is close to the algorithms we are going to learn.	***Because Yei's is just the same as the method I answered.***
Yu-Ting's Response	**Shiou-Chung's Response**

Figure 9. Students' responses to the task for developing students' critical thinking.

In the task, students were required to justify convincingly others' solutions and judge them by comparison to other methods. To this end, students first needed to read with comprehension others' written work and make sense of each mathematical symbol. Through the use of such kinds of assessment tasks, this process contributes to the development of students' critical thinking. For instance, the students, Yu-Ting and Shiou-Chung, gave the explanations of their preference as shown in Figure 10. Before answering it, they needed to read each method carefully and understand it. According to Yu-Ting's responses to the three different methods, she was able to interpret the meaning of each algorithm with mathematical thinking. In her written work, she showed that:

In Yei's solution, Yei decomposed 145×86 as 1 hundred 4 tens and 5 ones multiplied by 80 and 6. In Yun's method, Yun decomposed 145×86 as 1 hundred 4 tens and 5 ones multiplied by 86 ones. In Jean's method, Jean decomposed 145×86 as 145 ones multiplying 8 tens and 6 ones.

One the contrary, Shiou-Chung's responses to the assessment task were irrelevant to mathematics thinking. Shiou-Chung had difficulty understanding two other methods. As a result, she could not make a reasonable judgment about Jean's method.

Although the quality of students' critical thinking was not achieved as well as Jen-Jen's expectations, it was a good start for students to learn to justify others' ways of thinking. This task helped other teachers rethink what other task could be used for justifying other's way of thinking. Ying-Ying reflected on the use of assessment tasks in her weekly journal and described that:

> When comparing to my initial thought in the initiation of the study, I gradually agreed that mathematics is not simply memorizing rules and procedures. I also realized to create a classroom community with encouraging students to represent, talk, and listen to facilitate meaningful leaning mathematics. I have never known that developing students' critical thinking can be achieved by the use of assessment tasks until Jen-Jen shared with us her students' responses to the critical-thinking assessment tasks. I found that the task helped students to learn other ways to think about the meaning of the algorithm of three-digit multiplication and then constructed place value knowledge.

In addition, Mei-Mei reflected on the effect of the assessment tasks requiring students' to compare various solutions being discussed in the classroom on students' critical thinking in the interview conducted at the end of the study. She stated that the critical-thinking assessment tasks could be used to develop students' inter-assessment for students. She described that:

> I realized that students could learn from each other as the manner of the teachers' mutual learning. I exhibited each student's response on the wall of the classroom and asked my students' to read. I intended to develop their critical thinking through this way. However, students hardly read others' solutions in so many pieces on the wall at a time. I tried many different ways to improve it but it did not work very well. After I read the assessment task Jen-Jen generated, I then started to put two or three students' solutions into the assessment task for making a judgment. The effect of the assessment task on students' inter-assessment was imagined. I also realized classroom discourse is the major source of generating such kind of assessment task.

Improving Teachers' Awareness of Where Students Need to Make a Remedial Instruction. The task Yo-Yo generated was to examine students' understanding of the angle in which they engaged in a three-day lesson. The objective of the first day's lesson was to enable students to draw a rotated angle and compare the size of two angles. To achieve the objective, Yo-Yo asked each student to draw a rotated angle on his or her own whiteboard, then asked them to compare in pairs and determine which of the angles is bigger. As observed, midway through the task, she asked students to stop their work since most of the students did not draw an angle by rotating from an initial line to the end line. She said:

When drawing a rotated angle, you need to start with an initial line and rotate it counter-clockwise or clockwise instead of drawing an angle formed by two sides without rotation.

As observed, she was extremely concerned about the steps of drawing a rotated angle and labeling it to indicate the direction of rotation. Yo-Yo generated an assessment task to examine if students are able to compare the size of two angles as the prompt of students' mathematical journals at the end of the first day's lesson. She learned from professional dialogues in the weekly meeting about students' common misconception of angles determined by the length of two sides of a given angle that the longer side has the larger size of angle. The assessment task was shown in Figure 10.

As usual, Yo-Yo has completed the analysis of students' responses prior to bringing them to the weekly meeting. She found that ten out of 31 students in the class answered incorrectly as "*Angle A is bigger than B.*" She further summarized students' explanations of their incorrect answer. Their explanations were based on the following reasons: "*The two sides of angle A are longer. The angle A is wider. The area of angle A is bigger. It looks like. It was measured by the triangle ruler.*" After reviewing their students' responses, Yo-Yo was surprised with the students' misconceptions. Students determined the size of an angle by its irrelevant properties such as the length of two sides and its area of the angle. She put students' misunderstanding of the angle into her second day of the lesson.

In the second lesson, Yo-Yo accepted Mei-Mei's suggestions of using an overhead for a protractor projected on the blackboard in order to draw students' attention. She also prepared a protractor copied on a

Task: Po-Po drew two angles by using a triangle ruler. He is unable to compare which of the angles is bigger? Would you please to tell him which one is bigger? Give him a reason why it is.

Figure 10. The task of comparing two angles.

transparency for each student. The objective of the second lesson was to make her students recognize the structure of a protractor and measuring skill of an angle. In the first part of the second lesson, Yo-Yo asked each student to draw a rotated angle on the transparency protractor and remind students to label the sign for signifying the direction of the angle. Afterwards, she posted three students' incomplete work on the board including the two incomplete sides of the angle, no label for the size of the angle, and no sign for the direction of the rotation. She asked other students to fix the incomplete solutions and then continued the rest of the lesson.

The classroom discourse on mathematical ideas became the major source of conducting such kind of assessment task for diagnostic instruction. After the second lesson, Yo-Yo intended to examine if individual students truly understood the size of the angle. Therefore, she generated a task with two labels marked in two different locations for the angle as seen in Figures 11–13 as the prompt of the written task.

Before the third day of the lesson, Yo-Yo collected and analyzed students' responses to the task. The analysis of the whole class performing in the task is depicted in Table VI. Yo-Yo found that 39% of her students had difficulty with identifying the size of an angle. According to students' responses, Yo-Yo perceived that students misunderstood the size of angle either as "*the distance measured between two lines*", as Yi-Jer's writing in Figure 11 or as "the distance from the vertex to the label of the angle," as shown in Wen-Huei's explanation in Figure 12. As a consequence, they answered incorrectly with "*angle B is larger than angle A*." Thus, Yo-Yo recognized that there is a need to help students correct their misconception of an angle during the next day's lesson.

Yo-Yo brought students' responses to the following weekly meeting and shared some of students' misunderstanding of an angle. Jen-Jen responded to the assessment task and said that:

> Based on my teaching the same lesson, my students' understanding of an angle was disrupted by the lengths of two sides. When I asked them to give me a definition of an angle, they defined an angle as the area formed by the two lines. Such a misconception also appeared in Yo-Yo's class. In my previous lessons, I paid little attention to the dynamic process of producing an angle via the movement of rotation. However, I still do not know how to help those who had the difficulty with understanding the concept of an angle. I need to observe Yo-Yo how she made a remedial instruction for those students.

At the very beginning of the third day's lesson, the instructor, Yo-Yo, asked Yi-Jer and Su-Ting, who had the same incorrect response to the task, to come to the front of the classroom to explain their wrong answer.

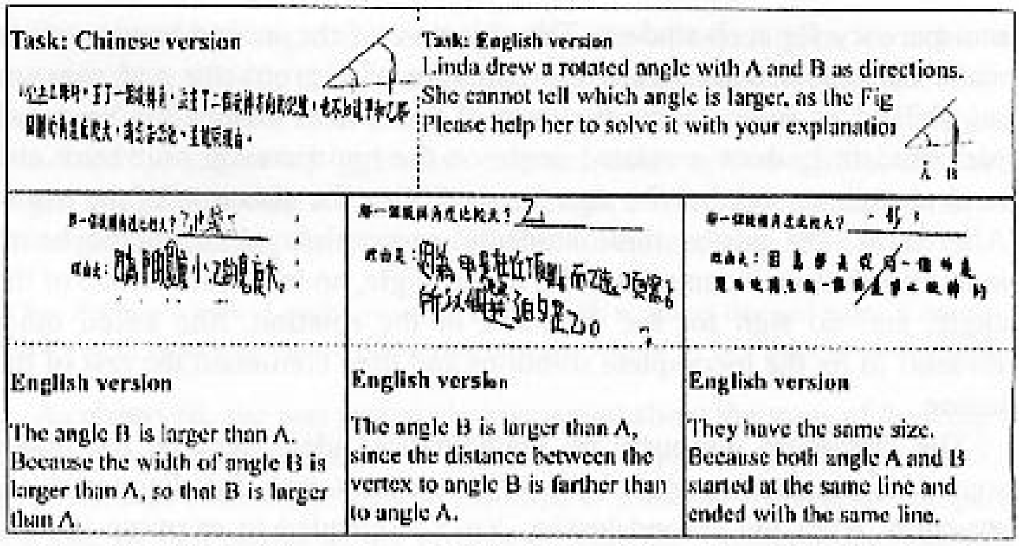


Figure 11. Yi-Jer's writing. *Figure 12.* Wen-Hue's writing. *Figure 13.* Dai-Jing's writing.

Table VII is a vignette of the instructor interacting with Yi-Jer, Su-Ting, and other students in the classroom.

The rest of the third lesson was to be continued. The assessment task evoking students' misconception of the concept of an angle was apparently enabling teachers to make immediate remediation during the next day's lesson. Thus, correcting students' misconceptions became the common work for the teachers at the very beginning of the daily lesson. As a result, this contributed to the teachers optimizing the quality of assessment and instruction, and thereby optimizing the learning of the students.

TABLE VI

An analysis of 31 students performing in the written tasks requiring students to understand the meaning of an angle

Students' various solutions		Percentage of students
Incorrect answer	1. Two sides of angle A are longer.	9%
	2. The distance measured between two sides of the angle A is wider.	9%
	3. The distance from the vertex to the label of the angle.	6%
	4. The area of angle A is bigger.	12%
	5. It looks like.	9%
	6. Others.	6%
Correct	7. They started at the line and stopped at the same line.	46%

TABLE VII

Vignette of students' responses to the assessment task informing remedial instruction

Yo-Yo	Tell us what you think, Yi-Jer.
Yi-Jer	Here (pointing to the label of A) [bracket ( ) here are added for clarity] is the width of angle A, and here (pointing to the label of B) is the width of angle B. So, the angle B is larger than angle A.
Sue-Ting	The angle B is larger, because of the width of angle A is just a little bit, and the width of B is much longer.
Yo-Yo	Is her thinking same as Yi-Jer's.
All	Yes.
Yo-Yo	Any question for them? Do you agree what they said? Raise your hand if you agree. Raise your hand if you disagree. (many hands are waving) [brackets ( ) here are added for clarity] (Kung-Yeu came to the front of the class and drawing another label for the angle) [brackets ( ) here are added for clarity]
Yo-Yo	What's your reason? Could you mark it on the figure?
Yo-Yo	Do both of you disagree that the arrow at A can be moved into there? Yi-Jer and Sue-Ting?
Yi-Jer and Sue-Ting	No.
All	The arrow cannot be moved to that place. They are the same angle.
Yo-Yo	Do you think they are the same angle? Yi-Jer. (Yi-Jer and Sue-Ting are still standing in the front of the class for the following discussion)
Fong-Cheng	Why can the angle B move to there? Is the angle B larger than A since they started in the same line?
Yi-Jer	Because the arrow of A is shorter than B.
Dai-Jing	Both A and B started at the same line and stopped at the same terminal line. Why can you say that B is longer than A?
Yo-Yo	Come to stand in the front of us and show us what you mean by that, Dai-Jing. (Dai-Jing rotated an angle from an initial line on the blackboard by using a plastic stick)
Bing-Hong	(Drawing another curl arrow on the angle on the board) If the arrow marked in B is longer, then the arrow at A can be changed into the curled line and straightening it; as a result, it is possible that the angle A is larger than the angle B. (students are laughing)
Yo-Yo	What do you mean?
Bing-Hong	The curled line becomes long line. Then the one is the longest.
Yo-Yo	Would you please to rotate it from A? Yi-Jer. Would you please to rotate it from B? Do they have the same size of the angle?
Yi-Jer	Yes.
Yo-Yo	Is the size of the angle determined by the distance between the two sides?

TABLE VII. Continued

Yi-Jer	No. (Yi-Jer and Sue-Ting's misconception has been corrected at this time)
Yo-Yo	Can we know dynamic process of an angle rotated without an arrow representing it?
All	No.
Yo-Yo	What can the arrow represent?
All	Direction of rotation.
Yo-Yo	Does one place turning to the other place refer to the direction or the size of the rotation?
All	Direction.
Yo-Yo	What can the direction tell us?
All	From where rotating where.
Yo-Yo	Is either from where to where or the degree of the rotation?
All	From where to where
Yo-Yo	What does the arrow represent?
All	Direction
Yo-Yo	Is the size?
All	Direction. (The class to be continued)

Discussions

The main conclusion of the study was that the use of the assessment task along with students' responses enhanced teachers understanding of students' learning and improved their reflective thinking of teaching when the assessment tasks were generated from daily lessons and supported by the researcher and same-grade teachers. The result is consistent with the literature research on the effect of assessment on informing further instructions (Webb, 2004). However, the assessment tasks referred to in previous studies are created either by researchers only (Amit & Hillman, 1999; Heuvel-Panhuizen, 1996), or by classroom teachers only (Lambdin et al., 1996). The process of generating assessment tasks involved in the study included observing teachers' same grade instruction, sharing what student mathematical learning occurred arising from the observations in weekly meetings, and group-work analysis of students' written work.

In group discussions at the weekly meeting, multiple perspectives and comments were shared in the process of generating assessment tasks. The issues discussed about students' learning became the candidates for developing the assessment task. Thus, the assessment tasks generated from same-grade teaching were centrally significant to teachers' attention and concerns. The number of tasks the teachers generated was increasing from time to time. Teachers' conceptions of students' learning, knowledge of assessment, and pedagogical practices were therefore expanded and deepened. This is the first feature of the study.

The assessment tasks which were reviewed by the researcher and same-grade teachers and implemented into other same-grade classrooms are more likely to enrich the purpose of tasks and broaden the variety of the assessment tasks. The intervention of the researcher placed more emphasis on theoretical perspectives of the assessment integral for instruction and students' mathematical learning, while the involvement of the teachers emphasized the practices of the assessment in classrooms. During the process of generating assessment tasks, the researcher's theoretical perspectives were examined against teachers' practices of assessment in the classroom. For instance, the researcher suggested teachers post students' responses to tasks on the wall of the classroom, but the suggestion did not work well through the teachers implementation in classroom. Thus, generating assessment tasks along with analyzing students' responses creates the possibility of connecting theory with practice and integrating assessment with instruction. This is the second feature of the study. Moreover, supporting teachers by professional dialogue for designing and using assessment tasks was as a means of improving the quality of assessment tasks. The result is consistent with Senk, Beckmann & Thompson, (1997) and Clarke's (1996) findings.

The use of the assessment tasks referred to in the study including generating assessment tasks along with students' responses to the tasks conceptualized teachers' professional learning and also improved students' mathematical learning simultaneously. The effects of the use of assessment tasks on conceptualizing teachers' professional learning included improving teachers' awareness of students' various solutions and learning difficulties to a specific problem, facilitating teachers' awareness of the importance of developing students' critical thinking, and improving teachers' awareness of where students need to make a remedial instruction. The problem-posing tasks were to improve students' ability to formulate problems, the communication tasks were to facilitate students' communicating about mathematics, while the problem-solving tasks were for teachers to help students making sense of their learning. Critical-thinking tasks were to facilitate students' critical thinking. The design of the use of assessment tasks for the study meets the principles of designing and planning a professional development program suggested by Loucks-Horsley et al. (1998). This is the third feature of the study.

The professional development referred to in the study included both individual teacher and organization development. The individual teacher development included the change of teachers' knowledge of students' learning and teachers' self-awareness. The change of knowledge of student learning was inspired by designing assessment tasks in line with analyzing students' responses to the tasks. Self-awareness refers to making sense of mathematics teaching and gaining their conceptions of learning. The organizational development included the organization of same-grade classrooms and school. This study suggests that the investigation of research on school-based teachers' professional development cannot be isolated from school context. The teachers involved in the research interacted closely with their same-grade colleagues, administrators, and parents of the school students. The teachers had the opportunity to bring the new ideas of teaching and learning to their colleagues, and then make the school culture change. In addition, teachers are able to transfer and generalize their knowledge and ability to other same-grade classrooms. This is the fourth feature of the study.

Finally, the findings of the study indicate that teachers with more teaching and research experience generated more and better quality assessment tasks. The experienced teacher played the role of a pilot for generating assessment tasks and analyzing students' responses.

NOTE

[1] The textbook of Taiwan dealing with "6 sets of 5 apples" as "six fives" or "five times six" represented as $5 \times 6 = (\ )$ is not consistent with those of other countries.

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